Amplitude Reduction of a Rotor Supported by

a Superconducting Magnetic Bearing Using Nonlinear Coupling Caused by Magnetic Force

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*Abstract*— In this paper, we propose a system in which a couple of movable superconducting bulks (SCs) support a rotor. Around the critical rotational speed, we investigated the whirling amplitude of the rotor in the system by theoretical, numerical and experimental methods. From analytical model, we derived governing equations of the system and predicted that internal resonance between the whirling rotor and the SCs can occur by nonlinear coupling. Numerical results indicated that internal resonance can occur and that the resonant whirling amplitude of the rotor can be reduced in the system. Finally, we verified these results by experimental method. The experimental results show that the whirling amplitude can be reduced when the SCs are swinging, confirming the maximum whirling amplitude reduction of the rotor utilizing internal resonance caused by nonlinear coupling.

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*Index Terms*— Electromagnets, High-temperature superconductors, Nonlinear dynamical systems, Rotors.

# I. Introduction

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igh-Tc superconducting magnetic levitation has some features such as low damping and high energy efficiency due to its stability without control. One of its applications is superconducting magnetic bearings (SMBs) [1]-[7]. However, SMBs tend to make the whirling amplitude of a levitated body larger at around the critical rotational speed by their low damping because of no physical contact. Further, complicated vibrations can be caused by the nonlinearity of the magnetic force. Therefore, it is necessary to reduce the whirling amplitude with considering effect of the nonlinearity on dynamics in passing through the critical rotational speed in applications. One of solutions for this subject is use of an internal resonance [8]-[11] between multiple oscillation modes coupled nonlinearly. When an internal resonance occurs, the kinetic energy can be exchanged between those modes and the amplitude of one of the modes can be reduced. Therefore, we suggested a system using SMBs in which the superconducting bulks can move in order to generate an internal resonance. In this study, we investigated amplitude reduction of the rotor.

Concerning the solution utilizing internal resonance, R. Kawana, *et al*. investigated motion of a rigid rectangular-shaped body having a rotor with unbalance and connected with three springs [12]. They performed nonlinear analysis of the rotor with internal resonance caused by geometrical nonlinearity. In addition, they numerically and experimentally clarified that internal resonance can reduce the amplitude of the rotor. R. Sakaguchi, *et al*. investigated nonlinear dynamics of a superconducting magnetic levitation system that consists of two permanent magnets on a rigid bar and two superconducting bulks under the bar. They numerically and experimentally confirmed that at its natural frequency the amplitude of horizontal oscillation is excited and that of vertical one is reduced because internal resonance caused by magnetic nonlinearity transfers the kinetic energy from vertical oscillation to lateral one [13]. We considered the whirling amplitude of a rotor can be also reduced according to the above previous researches. Therefore, in this study, by theoretical analysis, numerical calculations and experiments, we investigated resonant amplitude reduction of a rotor supported by SMBs utilizing internal resonance.

# II. Analytical Model and Governing Equations

## A. Analytical Model

Fig. 1 shows our analytical model. A permanent magnet (PM) is supported from its lateral side by two superconducting bulks (SCs). Each of the SCs is combined with a spring in order to obtain restoring force. The SCs can move keeping a constant distance *R* from the initial position of the PM, which is the origin of the coordinate. Fig. 2 shows the coordinates and electromagnetic forces related to the PM and the SCs. We define the angle between the center axis of the moving SCs and the *y*-axis as *θ*. The restoring forces due to the springs are regarded as linear. We assume that the PM, rotating at angular velocity *ω*, can whirl in the horizontal direction *x*-*y* place without inclining. We introduce a new coordinate system O-*x*’*y*’, which has the same origin as O-*xy* and is obtained by rotating the coordinate axes of O-*xy* through the angle *θ* about O. Therefore, the relationships of the two coordinates are described as follows:

*x*’ = *x*cos*θ* + *y*sin*θ* (1)

*y*’ = −*x*sin*θ* + *y*cos*θ* (2)

## B. Evaluating Electromagnetic Force

One of the SCs located in the negative *y*’ area is defined as SC1 and the opposite SC as SC2. We evaluate the electromagnetic forces by the advanced mirror image method [14]. In this method, we assume two mirror images in each of the SCs. The first mirror image (MI1) is derived from the initial position of the PM (Mag1). The second mirror image (MI2) is derived from the present position of the PM (Mag2). MI1’s and Mag1’s magnetization vectors are oriented in the same direction, while MI2’s and Mag2’s ones are in the opposite direction. Thus, the moving PM (Mag2) is affected by these mirror images in each of the SCs. The theoretically evaluated electromagnetic force is described as follows:

 (3)

where the PM is treated as a magnetic dipole. *M* is the magnetization vector of Mag2, *M*\* is that of MI2, and *M*0\* is that of MI1. *μ*0 is the permeability of vacuum. *R*\* is the position vector from MI2 to Mag2, and *R*0\* is that from MI1 to Mag2. By utilizing (3), the electromagnetic forces of the PM and the SCs are derived:

 (4)

 (5)

 (6)

 (7)

 (8)

 (9)

Electromagnetic forces *F**x*’1 and *F**y*’11 interact between MI1 in the SC1 and the PM (Mag2). *F**y*’12 interacts between MI2 in the SC1 and the PM (Mag2). *F**x*’2, *F**y*’21 and *F**y*’22 for SC2 are also given in the same way.

## C. Governing Equations and Nondimensionalization

By using (4) to (9), the equations of electromagnetic forces are derived as follows:

 (10)

 (11)

 (12)

Here *m* is the mass of the PM, *c* and *c**θ* are the damping coefficients of our system, *e* is the eccentricity of the mass center, *I* is moment of inertia of our system, and *k**θ* is the product of the spring constant and the moment arm. In (10) to (12), the dot symbol denotes differentiation with time *t*. Relation of *F**y*’1 and *F**y*’2 are follows:

*F**y*’1 = *F**y*’11 + *F**y*’12 (13)

*F**y*’2 = *F**y*’21 + *F**y*’22 (14)

(10) to (12) show that the PM’s movement in the *x* and *y* directions and the SC’s swinging in the *θ* direction are nonlinearly coupled with each other. In order to clarify this nonlinear coupling, (10) to (12) can be expanded into Taylor series around the origin of the coordinates (O) up to the 3rd order terms as follows:

 (15)

 (16)

 (17)

where *k**i* (i=1,2,…,7) are coefficients of terms appearing in the expanded form of (15) to (17). In order to order the terms, (10) to (12) can be nondimensionalized by using the following relations:

 (18)

Nondimensional equations of motion of the PM and the SCs are obtained as below:

 (19)

 (20)

 (21)

where the asterisks denoting nondimensional variables are omitted for simple description. Here *γ* and *γ**θ* are the nondimensional damping coefficients, *ε* is the nondimensional eccentricity, and *ν* is the nondimensional rotational speed defined by the ratio of *ω* to *ω**y*. Other constants such as *k**θθ* and *k**θxy* are also nondimensional ones. In (19) and (20), it should be noted that the ratio of the natural frequencies of the rotor’s oscillation in the *x* direction *ω**x* to *ω**y* is theoretically 1 to 2.

## D. Internal Resonance

If some of natural frequencies show an integer ratio in a multi-degree-of-freedom system which has quadratic nonlinear coupling terms such as *xθ*, *yθ* and *xy*, internal resonance between corresponding motions can occur. When this phenomenon occur, the kinetic energy can be exchanged between these motions. In our system, we considered that by adjusting *k**θθ*, the ratio of the natural frequency of the rotor’s oscillation in the *y* direction *ω**y* to that of the SCs’ swing motion *ω**θ* can be 2 to 1. Therefore, arranging that *ω**θ* to *ω**y* is 1 to 2, it can be expected that internal resonance occurs by quadratic nonlinear coupling terms *xθ*, *yθ* and *xy* and that amplitude reduction of *y* can be achieved by energy transfer from *y* to *θ* caused by internal resonance.

# III. Numerical Calculation

We performed numerical calculation of (10) to (12) in nondimensional form by means of the Runge-Kutta method, taking nonlinear terms into consideration. We dealt with two different systems for comparison: (a) with the SCs unfixed with optimal springs that adjust the ratio of *ω**y* to *ω**θ* to be 2 to 1 and (b) with the SCs fixed. We calculated the amplitude of the rotor in the *y* direction and the angle of the SCs in the *θ* direction in the cases where the rotational speed was increased and decreased.

Fig. 3 shows frequency responses of *y* and *θ* obtained by increasing the rotational speed. It can be found that in (a) the amplitude of *y* is effectively reduced at around the critical speed compared with the amplitude of *y* in (b), while the amplitude of *θ* is resonantly excited in (a). Fig. 4 shows time histories and FFT spectra of *y* and *θ* at *v*=1.0. We can find occurrence of internal resonance, in which not only the rotor resonates in the *y* direction, but also the SCs resonantly swing in the *θ* direction at the natural frequency 0.5 which is different from the excitation frequency *v* (=1).

Fig. 5 shows frequency responses of *y* and *θ* obtained by decreasing the rotational speed and Fig. 6 shows time histories and FFT spectra of *y* and *θ* at *v*=1.0 in the case where the rotational speed was decreasing. The same features can be found as above.

From these numerical results, we confirmed that the whirling amplitude of the rotor supported by unfixed SCs can be reduced by utilizing internal resonance.

# IV. 0064FRyz3Experiment

## A. Experimental Setup

Fig. 7 shows our experimental setup. A stainless-steel rigid shaft was connected to a power output section. The PM was connected to the bottom of the rigid shaft. The PM was supported by two SCs. In order to be cooled by liquid nitrogen, each of the SCs was placed in a container made ​​of polycarbonate. Each container was connected to a spring in order to receive restoring force. The containers including the SCs were swinging smoothly by a bearing. Using two-dimensional laser displacement meters, we measured the whirl amplitude of the rotor in the *y* direction and the angle of the SCs in the *θ* direction.

## B. Experimental Results

Fig. 8 and Fig. 9 show frequency response curves of the whirl amplitude and the angle in the case where the rotational speed is decreased. The ratio of *ω**y* to *ω**θ* is near 2 to 1 in Fig. 8, while it is 2 to 1.6 in Fig. 9. Compared with Fig. 9, it is found that in Fig. 8 the SCs’ swinging in the *θ* direction is excited and, at the same time, the rotor’s amplitude in the *y* direction is reduced at around the critical speed. Fig. 10 and Fig. 11 show time histories and FFT spectra of *y* and *θ* at *v*=1.0, corresponding to Fig. 8 and Fig. 9, respectively. It is found that in Fig. 8 the frequency of the main component in the *y* direction is 1.0, while that in the *θ* direction is 0.5. Therefore, we experimentally verified that the amplitude reduction of *y* at around the critical speed is caused by internal resonance.

# V. Conclusion

In this study, we analytically, numerically, and experimentally investigated nonlinear dynamics of a rotor supported from its lateral sides by two superconducting bulks. From analytical evaluation of nonlinear electromagnetic forces acting on the rotor supported by a SMB, we derived the governing equations for dynamics of the system. From those equations, we predicted that, if optimal springs are attached to the superconducting bulks, internal resonance between whirling of the rotor and swinging of the SCs can occur by quadratic nonlinear terms of the electromagnetic forces. Further, we performed numerical calculation taking nonlinear terms into consideration. Numerical results show that the whirling amplitude can be reduced by internal resonance at around the critical speed. Finally, we also carried out experiments, which verified our numerical and analytical predictions of occurrence of internal resonances.

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